Implementing PDR in CPAchecker

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September 23, 2016

Outline

Outline

Original IC3

Overview

- IC3 : Incremental Construction of Inductive Clauses for Indubitable Correctness
- Also known as PDR : Property Directed Reachability
- Aaron Bradley : “SAT-Based Model Checking without Unrolling”, VMCAI 2011
- Symbolic model checking algorithm for finite state systems (bit-level)
- Based on SAT solving, (relative) inductivity, backward analysis
- No unrolling of transition relation needed
- Highly incremental - lots of small SAT-queries
- Quickly became a staple part in most modern model checkers
- Adapted to infinite state systems such as software (C-programs, . . .)

Inductive Strengthening

- Property is inductive $\Rightarrow$ property is invariant
- But : Not every invariant property can be proved by induction

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- Idea : Strenghten property
- Math example : $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$ vs. $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$

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- Math example : $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$ vs. $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$
- Plan : Create strengthening of property and prove it by induction
- This will prove the property

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Preliminaries

Literal/Clause/Cube

- A *literal* is a propositional variable or its negation (x, $\neg y$, $\ldots$)
- A *clause* is a disjunction of literals ($x \vee \neg y$)
- A *cube* is a conjunction of literals ($x \wedge \neg y$)
- Therefore, the negation of a cube is a clause ($\neg(x \wedge \neg y) \equiv (\neg x \vee y)$)

Transition System

A *Transition System* $S : (\bar{x}, I(\bar{x}), T(\bar{x}, \bar{x'}))$ consists of

- a set $\bar{x}$ of state variables
- the initial configuration of the system $I(\bar{x})$
- the transition relation $T(\bar{x}, \bar{x'})$

Preliminaries - Cont.

(Relative) Inductivity

Given a transition system $S : (\bar{x}, I(\bar{x}), T(\bar{x}, \bar{x'}))$:

- P is inductive, if $I \Rightarrow P$ and $P \wedge T \Rightarrow P'$
- P is inductive *relative* to F, if $I \Rightarrow P$ and $F \wedge P \wedge T \Rightarrow P'$

Safety property : P

A boolean formula that is always true for a given transition system

Inductive Strengthening

An inductive strengthening of a safety property P is a formula F, so that $F \wedge P$ is inductive

Preliminaries - Cont.

State

Assignment of values to **all** state variables of the transition system.
Represented by a cube

Control Flow Automaton (CFA)

A *Control flow automaton* $A = \{L, G, l_0, l_E\}$ consists of

- a set of locations $L = \{0, \ldots, n\}$ representing the program counter
- edges from $G \subseteq L \times QFFO \times L$ labeled with quantifier-free first order formulas describing the transition
- an initial location l_0
- an error location l_E

Outline

Original PDR

General Concepts

Frame : F_i

- Overapproximation of reachable states in at most i steps from initial states
- Set of clauses (act as constraints regarding reachability)
- As formula : conjunction of clauses (CNF)

Counterexample to Inductiveness : CTI

State that can reach a non-property state (bad state) in one or more steps

Original PDR

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Counterexample to Inductiveness : CTI

State that can reach a non-property state (bad state) in one or more steps

- Maintain series of stepwise overapproximations $F_0, \ldots, F_k$ for increasing k
- $F_0 = I$ and initially $F_i = P$ for $i \neq 0$ (assume P is invariant)
- Continuously refine frames by adding reachability information
- Derived from recursively backward-analyzing CTIs

Original PDR

General Concepts

Basic Invariants

- $F_0 \Leftrightarrow I$
- $F_i \Rightarrow P, \quad \forall\, 0 \leq i \leq k$ - "every frame satisfies P"
- $F_i \Rightarrow F_{i+1}, \quad \forall\, 0 \leq i < k$ - "every F_{i+1} is more general than F_i"
  $clauses(F_{i+1}) \subseteq clauses(F_i)$
- $F_i \wedge T \Rightarrow F'_{i+1}, \quad \forall\, 0 \leq i < k$ - "states in F_i transition to states in F_{i+1}"

Original PDR

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Basic Invariants

- $F_0 \Leftrightarrow I$
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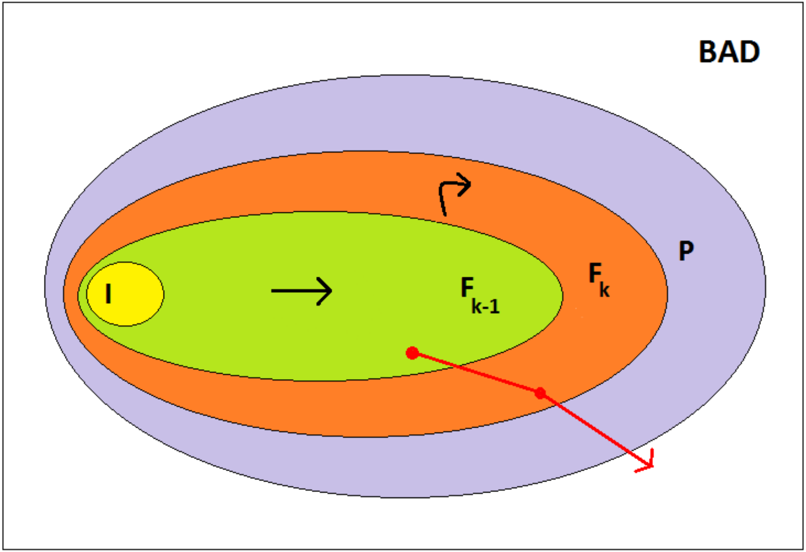
  $clauses(F_{i+1}) \subseteq clauses(F_i)$
- $F_i \wedge T \Rightarrow F'_{i+1}, \quad \forall\, 0 \leq i < k$ - "states in F_i transition to states in F_{i+1}"

$\Rightarrow$ Fixpoint reached if $\exists\, i$ so that $F_i = F_{i+1}$

$\Rightarrow$ Property holds

$\Rightarrow$ F_i is an inductive strengthening of the safety property P

PDR : Identify CTIs



Algorithm

check for 0-/1-step counterexample ($I \wedge \neg P$ / $I \wedge T \wedge \neg P'$)

for $k = 1$ to ...

- while (CTI exists $\equiv F_k \wedge P \wedge T \Rightarrow P'$ not true)
  - get CTI **s** from model
    - % Blocking Phase %
    - prove **s** is unreachable in $\leq$ **k** steps
      (this is where new clauses are learned)
    - if not possible $\rightarrow$ error found
- % Propagation Phase %
- for $i = 1$ to k and all clauses **c** in F_i
  - if **c** became inductive
    $\equiv F_i \wedge c \wedge T \Rightarrow c'$ is true : add **c** to F_{i+1}
- if $\exists\ i$ so that $F_i = F_{i+1} \rightarrow$ property holds

Blocking a state s at F_i : Proof Obligation (s, i)

Block state s at F_i $\equiv$ Prove s is unreachable in $\leq i$ steps

- If $i = 0$ and s intersects with initial states $\rightarrow$ error found
- Check : $\neg s$ inductive relative to $F_{i-1} \equiv F_{i-1} \wedge \neg s \wedge T \Rightarrow \neg s'$ is true
- No : try to block predecessor p of s at F_{i-1} first (DFS).
  Add **Proof Obligations** $(p, i-1)$ and (s, i)
- Yes : add $\neg s$ to all frames $F_1, \ldots, F_i$. Also add PO $(s, i+1)$ if $i < k$
- Pick PO with lowest frame number next
- Retry previously failed attempts until s could be blocked at F_i

Algorithm : Important Improvements

Generalization

- Blocking one state s at a time is ineffective
- When adding $\neg s$ at level i : find $c \subseteq \neg s$ that is still inductive and add c instead
- c may exclude many more states than $\neg s$ $[(\neg x \vee y) \rightarrow \neg x]$
- Drop literals that don't actually contribute to result of induction query $F_{i-1} \wedge \neg s \wedge T \Rightarrow \neg s'$
- Use unsat-core, ternary simulation, ...

Algorithm : Important Improvements

Lifting

- Similar intention as with generalization
- When computing a predecessor p of state s : find **set** of states that also transition to s
- Represented by a sub-cube of p

Subsumption

- Suppose $F_i = \{s\}$ with $s = x \vee y$ and we can add $\hat{s} = x$
- Note that $\hat{s} \Rightarrow s$ (or alternatively $literals(\hat{s}) \subset literals(s)$)
- s doesn't represent more reachability info than $\hat{s}$
- Simply remove s
- Avoids redundancy and keeps frames small (easier SAT-queries)

Outline

PDR on Control Flow Automata

Based on “IC3 Software Model Checking on Control Flow Automata” by T. Lange et al.

- Apply PDR directly to CFA
- Use SMT-solver instead of SAT-solver
- Check reachability of error location
- Use single transitions between locations (no unrolling needed)
- Create frames $F_0, \ldots, F_k$ for **every** location
- Represents k-step reachability for this location, starting at l_0

PDR on Control Flow Automata

Based on “IC3 Software Model Checking on Control Flow Automata” by T. Lange et al.

PDR Relative Inductivity Check

When trying to block a state s at level $i \equiv (s,i)$

- $F_{i-1} \wedge \neg s \wedge T \wedge s'$ (meaning : $F_{i-1} \wedge \neg s \wedge T \Rightarrow \neg s'$)

Adjusted Relative Inductivity Check

When trying to block a state s at location l at level $i \equiv (s, l, i)$

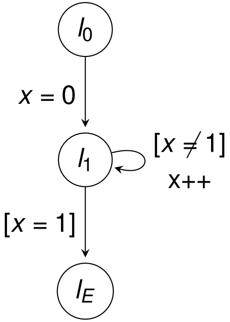
- Case 1 : $F_{i-1,l\_pred} \wedge T_{l\_pred \rightarrow l} \wedge s'$, if $l \neq l\_pred$
- Case 2 : $F_{i-1,l\_pred} \wedge \neg s \wedge T_{l\_pred \rightarrow l} \wedge s'$, if $l = l\_pred$

where l_pred is a predecessor location of l.

- Unsat $\rightarrow$ add $\neg s$ to all $F_{j,l}$ where $j \leq i$
- Sat $\rightarrow$ get predecessor state p and add POs $(p, l\_pred, i-1)$ and (s, l, i)

IC3CFA

Example



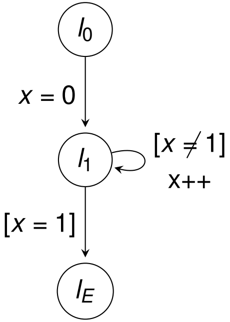
Initialization

- No 0-/1-step counterexamples
- $F_{i,l} = true$, for all locations l and levels i (we have no known safety property!)
- Except : $F_{0,l} = false$, for all non-initial locations

loc / lvl	0	1	2
l_0	true	true	true
l_1	false	true	true

IC3CFA

Example



First iteration : $k = 1$

- Transition still possible?
- $F_{1,l_1} \wedge T_{l_1 \to l_E} =$
  $true \wedge x = 1 : SAT$
- $\rightarrow x = 1$

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IC3CFA

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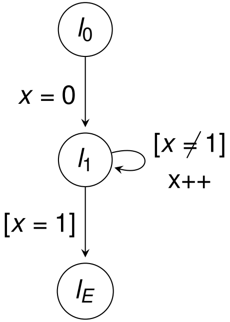


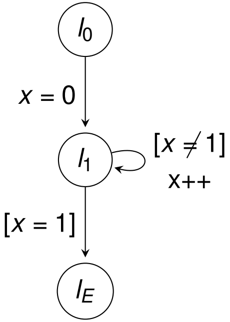
First iteration : $k = 1$

- Try to block $x = 1$ at l_1 at level 1
- **Predecessor** l_0 :
- $F_{0,l_0} \wedge T_{l_0 \rightarrow l_1} \wedge s' =$
  $true \wedge x' = 0 \wedge x' = 1$: $UNSAT$
- $\rightarrow$ add $x \neq 1$ to F_{1,l_1} and F_{0,l_1}

loc / lvl	0	1	2
l_0	true	true	true
l_1	false, $x \neq 1$	true, $x \neq 1$	true

IC3CFA

Example



First iteration : $k = 1$

- Try to block $x = 1$ at l_1 at level 1
- **Predecessor** l_1 :
- $F_{0,l_1} \land \neg s \land T_{l_1 \to l_1} \land s' = (false \land x \neq 1) \land x \neq 1 \land (x \neq 1 \land x' = x + 1) \land x' = 1 : UNSAT$
- $\rightarrow$ add $x \neq 1$ to F_{1,l_1} and F_{0,l_1}

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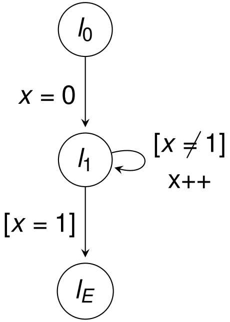

First iteration : $k = 1$

- Transition still possible?
- $F_{1,l_1} \wedge T_{l_1 \to l_E} =$
  $(true \wedge x \neq 1) \wedge x = 1$: *UNSAT*
- Termination? $\to$ No
- $\to$ continue with next iteration

loc / lvl	0	1	2
l_0	true	true	true
l_1	false, $x \neq 1$	true, $x \neq 1$	true

IC3CFA

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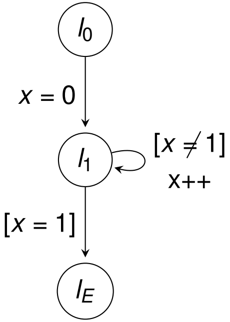


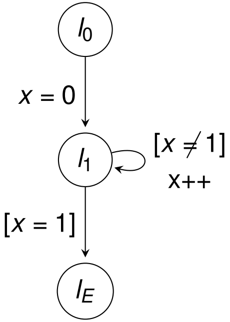
Second iteration : $k = 2$

- Transition still possible?
- $F_{2,l_1} \wedge T_{l_1 \to l_E} =$ $true \wedge x = 1 : SAT$
- $\to x = 1$

loc / lvl	0	1	2
l_0	true	true	true
l_1	false, $x \neq 1$	true, $x \neq 1$	true

IC3CFA

Example



Second iteration : $k = 2$

- Try to block $x = 1$ at l_1 at level 2
- **Predecessor** l_0 :
- $F_{1,l_0} \wedge T_{l_0 \rightarrow l_1} \wedge s' =$
  $true \wedge x' = 0 \wedge x' = 1$: *UNSAT*
- $\rightarrow$ add $x \neq 1$ to F_{2,l_1} and F_{1,l_1} and F_{0,l_1}

loc / lvl	0	1	2
l_0	true	true	true
l_1	false, $x \neq 1$	true, $x \neq 1$	true, $x \neq 1$

IC3CFA

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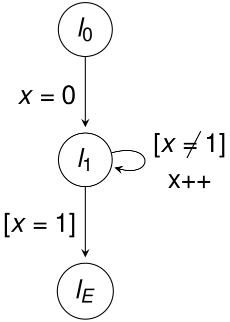


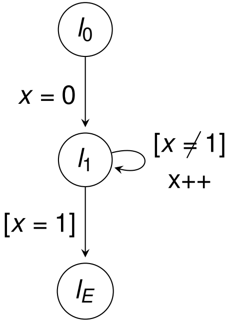
Second iteration : $k = 2$

- Try to block $x = 1$ at l_1 at level 2
- **Predecessor** l_1 :
- $F_{1,l_1} \wedge \neg s \wedge T_{l_1 \to l_1} \wedge s' = (true \wedge x \neq 1) \wedge x \neq 1 \wedge (x \neq 1 \wedge x' = x + 1) \wedge x' = 1$: SAT $\to x = 0$
- Proof-obligations : $(1, l_1, x = 0)$, $(2, l_1, x = 1)$

loc / lvl	0	1	2
l_0	true	true	true
l_1	false, $x \neq 1$	true, $x \neq 1$	true, $x \neq 1$

IC3CFA

Example



Second iteration : $k = 2$

- Pick lowest Proof-obligation $(1, l_1, x = 0)$
- **Predecessor** l_0 :
- $F_{0,l_0} \wedge T_{l_0 \to l_1} \wedge s' =$ $true \wedge x' = 0 \wedge x' = 0$: *SAT* $\to x = 0$
- Proof-obligations : $(0, l_0, x = 0)$, $(1, l_1, x = 0)$, $(2, l_1, x = 1)$
- Next : $(0, l_0, x = 0) \to$ Error found !

loc / lvl	0	1	2
l_0	true	true	true
l_1	false, $x \neq 1$	true, $x \neq 1$	true, $x \neq 1$

Remark : Dealing with infinite state space

Weakest Preconditions

- Use weakest preconditions on local transitions to calculate exact predecessors
- Can be expensive for large transitions

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Weakest Preconditions

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Predicate Abstraction

- Get concrete predecessors from model of SAT-query (like original PDR)
- Apply predicate abstraction and work with abstract states
- Random example : $(x = 0 \land y = 0) \rightarrow x = y$
- When finding abstract transition with no concrete counterpart
  $\rightarrow$ abstraction was too broad
  $\rightarrow$ interpolate and refine abstraction ($x = y \rightarrow (x = y \land x \geq 0)$)
- Similar to CTIGAR

Implementation in CPAchecker

Transitions

- CPAchecker can be configured to arbitrary block size
- Large Block Encoding currently used for PDR
- PredicateCPA used to get path formulas of edges between locations

Predicate Abstraction

- Component PredicateAbstractionManager of PredicateCPA used for computing abstraction based on current predicates
- SMT-solver used to get interpolant that leads to new abstraction predicate

Outline

Summary

- PDR is a symbolic model checking algorithm for finite state systems based on SAT-solving, relative inductiveness, inductive strengthening
- Blocking phase : Identify CTI and recursively block it
- Propagation phase : Push clauses to next frame if they became inductive after blocking phase
- PDR can be extended to infinite state systems in multiple ways
- One way : Apply PDR directly to CFA (IC3CFA)
- Give every location its own set of stepwise overapproximations (frames)
- Check reachability of error location using single transitions between locations

Outlook

What still needs to be done

- Predicate abstraction
- Check if it pairs well with location local frames

For the future

- One prover environment for each frame
- Keep frame clauses on prover stack (exploit incremental nature of PDR)
- Parallel implementation (PDR is suitable for this)